Mathematical Modeling of Wind Energy System Using Two Mass Model Including Generator Losses

Patil Ashwini, Dr. Thosar Archana

Abstract: Evolution of new technology has increased the power demand. Because of such high demands and limited energy resources it is essential to optimize the energy production and cost. The energy companies are leading strong researches of the wind energy systems for energy security purpose.

Large capacity wind energy systems are installed offshore where non planned service is very costly. The fault tolerant control systems are beneficial when faults are detected during next planned service. It secures the energy production even if fault occurs. For designing any control system an appropriate mathematical model is always needed. The literature reveals one mass, two mass, three mass and six mass mathematical models of wind energy system. An attempt is made to develop a modified two mass mathematical model to design a fault tolerant control system. In wind energy systems the mechanical faults like misalignments in the drive train, gears and bearings faults are very frequent. These are subject to a wear process and cause frictional losses. This paper addresses these faults in the mathematics of the wind energy system. Further the work is extended to study the variations of the parameters generator inertia constant, spring constant, viscous friction coefficient and gear ratio on the pole-zero plot and is related with the physical design of the wind turbine. Behaviour of the wind turbine during drive train faults are simulated and briefly discussed.

Keywords: Mathematical model of wind energy system; pole-zero plot; stability analysis; shaft stiffness; viscous friction coefficient; gear ratio; generator inertia

I. INTRODUCTION

The conventional energy sources use fossil fuels. It has adverse effect of polluting environment. The problem of global warming and reduction of fossil fuel resources has forced to look for the alternative energy resources. The increased government support has caused the flourished wind power installation globally. Therefore, the control engineers have challenge to develop the techniques for reducing the operational cost and increasing the reliability. In last decades the wind energy systems have been a subject of an intense research programme.The focus is on large capacity wind energy systems. The large capacity wind energy systems are remotely located. The frequent fluctuations and large change in intensity of wind causes output power fluctuations. It also causes various faults in the whole structure, which may result the major failure of the system. To study the impact of these faults and design an advance fault tolerant control systems the wind energy system must be effectively mathematically modeled. Also its stability study must be done intensely.

In recent papers the research has been done on the modeling and simulation of the wind energy system. The detailed description of nonlinear wind turbine simulation model is given in [1]. The stochastic wind speed variations caused by turbulence, wind shear and tower shadow causes the 3 peak pulsations in aerodynamic torque and have effect on power quality [2]. In wind energy system the drive train is a very complex mechanical structure and susceptible to the faults cause by wind disturbance. The transient response of it is studied in [3], [4]. There are six mass, three mass, two mass and one mass model of the wind energy system in the literature [5]. The two-mass model can be effectively used with sufficient accuracy. The effects of drive train parameters such as inertia constant, spring constant, damping constant and gear ratio on transient response is studied in [6], [7].

The goal of this paper is to develop the mathematical model of wind energy system and analyze its stability by pole-zero plots. The mathematical model intends to use for designing fault tolerant control system in future. The drive train system represents the set of components necessary to transmit the power from the rotor to the generator. As the large capacity wind energy systems have become bigger and heavier, the components used are more flexible and deformable. It leads to the significant vibrations and highly varying stresses. It causes the frictional losses which need to be considered in the mathematical modelling. The aerodynamic torque and generator torque are the two inputs to the drive train from rotor side and generator side. The large torque produced in the drive train may result in torsion oscillations among different sections of the wind energy shaft system. In the severe cases, the drive shaft of a generator may crack and even break. The fatigue accumulation when bearing a large torque repeatedly may reduce the life of the shaft. Due to any fault in generator-converter, the drive train distortion may occur and results in the oscillations of wind turbine torque. The mathematical model derived in literature doesn't consider the torque loss due to the misalignment or the bearing faults which are very common. In this paper the torque loss is included in the mathematics so that the model should give results which should be more accurate to real.

II. NOTATION

The notation used throughout the paper are stated below.

V_r the rotor effective wind speed in [m/s]

$\beta(t)$ pitch angle in [^{o}]

$\beta(t)ref$ reference pitch angle in [^{o}]

T_a the aerodynamic torque in [N],

T_g	generator torque in [N]
w_r	rotor speed in [rad/s]
w_g	generator speed in [rad/s]
x_t	displacement of nacelle from equilibrium position [m]
ρ	the density of the air,
A	rotor swept area exposed to the air
C_p	power coefficient of a turbine, a function of pitch angle and tip speed ratio
C_t	thrust coefficient, a function of pitch angle and tip speed ratio.
ω	natural frequency
ζ	damping ratio of pitch actuator model
N_g	gear ratio
J_g	inertia of generator and high speed shaft in [Kgm^2]
J_r	inertia of rotor and low speed shaft
B_{dt}	viscous friction coefficient
K_{dt}	shaft stiffness in [Nm/rad]
M	mass of tower in [Kg]
K_t	tower spring coefficient in [Nm]
B_t	tower damping coefficient
τ	time constant of the generator system

III. WIND ENERGY SYSTEM MODELING

The wind energy system is represented by the Fig.1 includes the wind model, aerodynamics, pitch actuator, tower, drive train and generator. The wind model includes the effects of wind shadow, shear and turbulence. The aerodynamic model calculates the aerodynamic torque and thrust with the rotor effective wind. When wind speed is more than rated wind speed the pitch angle β is adjusted by the pitch actuator to maintain the rated rotor speed. The aerodynamic torque T_a is the input to the drive train. The drive train increases the speed of the rotor. Finally the generator model includes the generated power calculation.

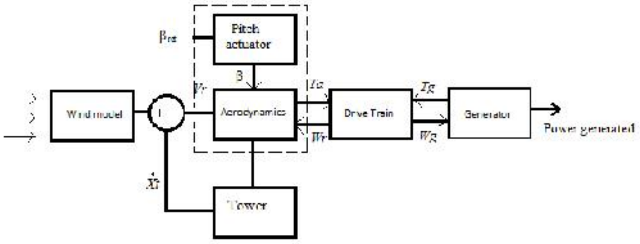

Fig.1 Wind energy system divided into sub-models

A. Wind Model

Wind speed variations due turbulence and periodic wind speed variations caused by wind shear and wind shadow leads to the dynamic loading of wind turbine. As the air passes over the earth, it causes friction with ground, buildings, hills, trees and other structures called wind shear. As the blades are rotated, when these are at lower side it experiences more wind shear. When the rotor effective wind speed is considered for individual blade, it decreases when blade comes in front of the tower. The wind flow is redirected due to the presence of the tower called wind shadow effect. The tower shadow is more dominant in determining the dynamic torque oscillations compare to wind shear. Wind turbulence depends on various environmental factors like temperature, pressure, humidity as well as the motion of wind itself in three dimensions. Fig. 2 shows that effective wind is the collective effect of wind shear, wind shadow and turbulence.

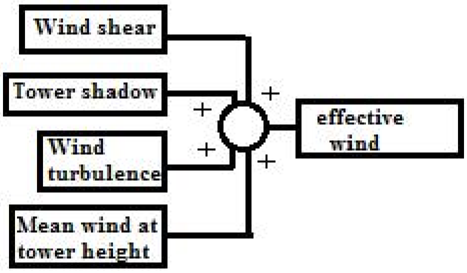

*Fig.*2 Effective wind considering wind shear, shadow and turbulence

B. Aerodynamic Model

Aerodynamic torque is transferred from the rotor to the generator through drive train. It is given by equation (1)

$$T_a = \frac{1}{2w_r}\rho A V_r^{\,3} C_p \tag{1}$$

The pitch angle can be changed to control the rotor speed. In case of higher wind speed the pitch angle has to increase to maintain the rated rotor speed. Any fault occurred in case of pitch angle position may cause the system to go in runaway condition which is the failure of the system.

In partial load region maximum value of C_p is maintained by adjusting very small pitch angle where the maximum power can be captured. In full load region, the rotor speed is maintained constant by adjusting the pitch angle and the rated power is maintained.

The wind acting on the rotor causes aerodynamic thrust F_{th} in [N] on the tower which makes the tower to sway back and forth. It is given by equation (2)

$$F_{th} = \frac{1}{2} A V_r C_t \tag{2}$$

C. Pitch Actuator:

Pitch actuator is used to adjust the pitch angle of blade to maintain the rated rotor speed. It is given by equation (3).

$$\ddot{\beta}(t) = -2\zeta\omega\dot{\beta}(t) - \omega^2\beta(t) + \omega^2\beta(t)ref \tag{3}$$

State space model for the pitch actuator is given by equation (4).

$$\begin{bmatrix} \dot{\beta}(t) \\ \ddot{\beta}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & -2\zeta\omega \end{bmatrix} \begin{bmatrix} \beta(t) \\ \dot{\beta}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -\omega^2 \end{bmatrix} [\beta(t)ref\,] \tag{4}$$

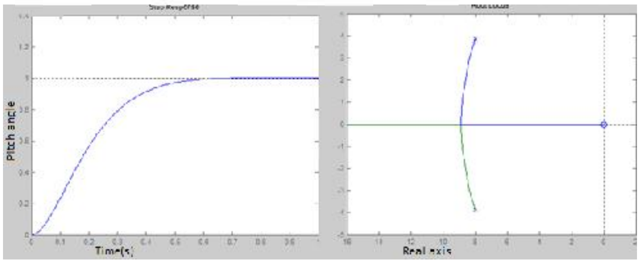

Fig.3 Step response and root locus for pitch actuator

Step response and root locus are plotted in Fig.3.

D. Drive Train Model

The drive train consists of the rotor, main gear box and the generator. The main gear box is used to increase the speed of low speed rotor shaft to high speed generator shaft. It is a complex mechanical structure and vulnerable to the faults. There are fluctuations in the aerodynamic torque from rotor because of variable input wind. The drive train modifies the torque transmitted. This modified torque can be assessed by analyzing drive train model response thoroughly.

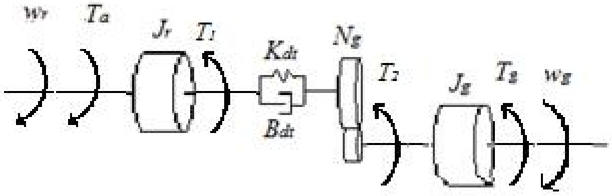

Fig.4 Drive train model

The drive train is divided into three parts as low speed shaft, gear box and high speed shaft which is connected to generator as shown in fig. 4. The two mass mathematical equations are written analyzing its response which is sufficiently accurate. The torsion spring is included to show the flexibility of drive train. The aerodynamic torque and the electrical torque are the inputs to the drive train form the rotor and the generator side. As they are opposite to each other, they cause power imbalances. The coupling shaft experiences a twist at every instant of power imbalance due to the variation in wind speed along with other associated parameters. It causes mechanical transients leading to sustained oscillations in energy conversion system. The differential equations take the form as

$$J_r\dot{w}_r = T_a - K_{dt}\theta_\delta - B_{dt}\dot{\theta}_\delta \qquad (5)$$

$$J_g N_g \dot{w}_g = -T_g N_g + K_{dt}\theta_\delta + B_{dt}\dot{\theta}_\delta \qquad (6)$$

$$\dot{\theta}_\delta = w_r - \frac{w_g}{N_g} \qquad (7)$$

The state space model of the drive train is formed as equation (8)

$$\dot{X} = AX + Bu$$
$$Y = CX.$$

Where,

$$A=\begin{bmatrix} \frac{-B_{dt}}{J_r} & \frac{B_{dt}}{J_r N_g} & \frac{-K_{dt}}{J_r} \\ \frac{B_{dt}}{J_r N_g} & \frac{-B_{dt}}{J_g {N_g}^2} & \frac{K_{dt}}{J_g N_g} \\ 1 & \frac{-1}{N_g} & 0 \end{bmatrix}, \quad B=\begin{bmatrix} \frac{1}{J_r} & 0 \\ 0 & \frac{-1}{J_g} \\ 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\text{with, } X = \begin{bmatrix} w_r \\ w_g \\ \theta_\delta \end{bmatrix}, u = \begin{bmatrix} T_a \\ T_g \end{bmatrix}, Y= \begin{bmatrix} w_r \\ w_g \\ \theta_\delta \end{bmatrix} \qquad (8)$$

To analyze the characteristics of the drive train, rotational speed of high speed shaft must be studied. To study it in detail the transfer functions of generator speed for two inputs aerodynamic torque, w_g/T_g and generator torque, w_g/T_a are analyzed by plotting its step responses, root locus. The aerodynamic torque is the input torque generated by the rotor with input wind. Generator torque is an opposing torque generated by the generator.

The step response, root locus for w_g/T_a and w_g/T_g are plotted in Fig.5a.

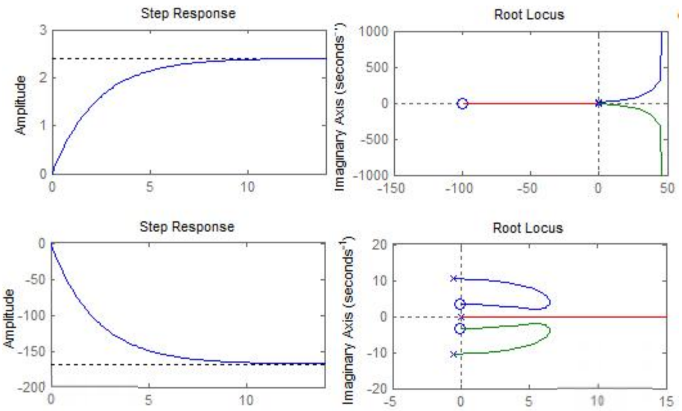

Fig.5a Step response and root locus for the transfer functions of w_g/T_a and w_g/T_g

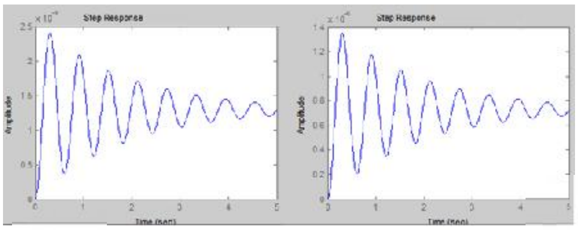

Fig.5b Closed look of Step response for the transfer functions of w_g/T_a and w_g/T_g

If we have a close look of the step responses of w_g/T_a and w_g/T_g plotted in Fig.5b, it is found that there are oscillations initially. Also the curves take more time to settle. These oscillations may cause the fatigue damage to the system when mechanical or electrical fault occurs. So any fault in a system may lead to wear and tear of mechanical part and also a major damage.

Looking at the pole-zero plot of the system in Fig.6, there are two dominant poles and third real pole which is near the origin in the left hand plane. The system is stable but takes more time to settle due to the pole very near the origin. Due to any mechanical or electrical fault, if pole moves at right hand plane, system will be unstable.

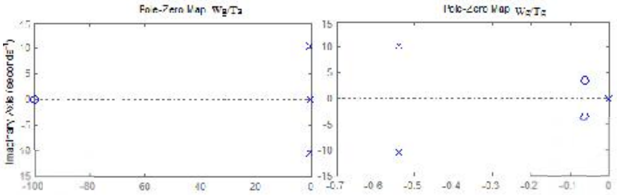

Fig 6.Pole –zero plot for the transfer functions of w_g/T_a and w_g/T_g

E. Tower:

The effective wind speed on the rotor causes thrust force in the direction of wind on tower. It causes tower acceleration and reduces the effective wind speed of rotor shown in Fig.7. The tower gets displaced from the equilibrium due to thrust by distance x_t. It causes the reduction of rotor effective speed. Mathematically the tower can be modeled as equation (9)

$$M\ddot{x}_t = F_{th} - K_t x_t - B_t \dot{x}_t \qquad (9)$$

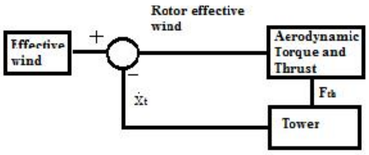

Fig.7 .Rotor effective wind speed affected by tower acceleration

State space function for tower is given by equation (10)

$$\begin{bmatrix} \dot{x}_t \\ \ddot{x}_t \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-K_t}{M} & \frac{-B_t}{M} \end{bmatrix} \begin{bmatrix} \dot{x}_t \\ \ddot{x}_t \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} [F_{th}] \qquad (10)$$

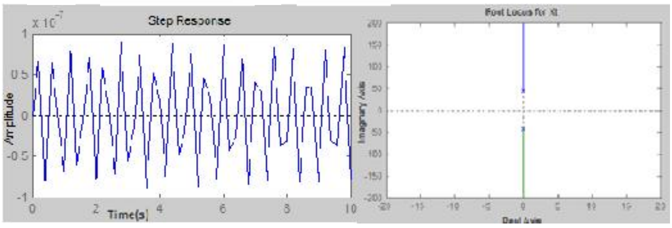

Fig.8 Step response and root locus for tower acceleration

The step response and root locus of tower acceleration to step change in F_{th} are plotted in Fig.8.

F. Generator:

Generator is a very important component in the wind energy system. It has to work under fluctuating power levels and tune with the wind variations. The various types of generator i.e. induction type, synchronous type and now a day popularly used doubly fed type induction generator is used. Electric power is generated by the generator, and to enable variable-speed operation, currents in the generator are controlled using power electronics. Therefore, power electronic converters interface the wind turbine generator output with the utility grid. The first order model of generator with converter can be represented by equation (11)

$$\frac{T_g}{T_{gref}} = \frac{\tau}{\tau s+1} \qquad (11)$$

The power produced by the generator depends on the rotational speed of the rotor and the applied load. It is described by the equation (12).

$$P_{g} = \eta_g W_g T_g \qquad (12)$$

where η_g is the efficiency of the generator.

IV. STABILITY ANALYSIS OF WIND TURBINE

Drive train is very important part of the variable speed wind energy system. It is very complex mechanical structure. The frequent mechanical faults in it are due to misalignment and damaged bearing. The misalignment causes non-uniform rotation. Damaged bearing due to improper lubrication and irregular maintenance causes increased friction. Providing the regular maintenance to especially offshore wind energy systems is very costly. Any such fault may cause the transients shown in Fig.9. The system finally stabilized with some steady state error in output generator speed.

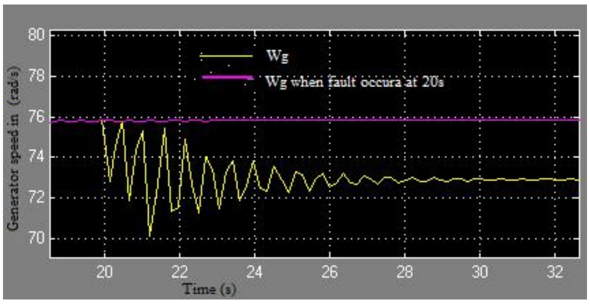

Fig 9. Generator speed when fault occur at 20 second

At variable rotational speed, these losses and faults vary strongly with torque transmission. In partial load region, the rotational speed is varying. Considering this type of fault the T_{loss} is additional term must be considered in the above model to design controller in future which can take care of steady state error. So equation (6) takes the form as equation (13)

$$J_g N_g \dot{w}_g = -T_g N_g - T_{loss} N_g + K_{dt}\theta_\delta + B_{dt}\dot{\theta}_\delta \qquad (13)$$

Further it is converted into the state space form as equation (14)

$$\dot{X} = AX + B1u + B2v$$
$$Y = CX.$$

Where,

$$A = \begin{bmatrix} \frac{-B_{dt}}{J_r} & \frac{B_{dt}}{J_r N_g} & \frac{-K_{dt}}{J_r} \\ \frac{B_{dt}}{J_r N_g} & \frac{-B_{dt}}{J_g N_g^2} & \frac{K_{dt}}{J_g N_g} \\ 1 & \frac{-1}{N_g} & 0 \end{bmatrix}, \quad B1 = \begin{bmatrix} \frac{1}{J_r} & 0 \\ 0 & \frac{-1}{J_g} \\ 0 & 0 \end{bmatrix}, \quad B2 = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-1}{J_g} \\ 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\text{with, } X = \begin{bmatrix} w_r \\ w_g \\ \theta_\delta \end{bmatrix}, u = \begin{bmatrix} T_a \\ T_g \end{bmatrix}, v = [T_{loss}], Y = \begin{bmatrix} w_r \\ w_g \\ \theta_\delta \end{bmatrix}$$

(14)

It is necessary to study the detail stability analysis of drive train system when disturbances occur. Four cases have been considered for detail transient stability analysis of drive train model from equation (14)

A. Case1: Change in viscous friction coefficient:

It is observed that viscous friction coefficient has significant effect on the stability. Here the mutual viscous friction between generator and rotor is considered.

If we analyze the pole-zero plot of the transfer function w_g/T_a, the real zero moves towards the dominant poles as B_{dt} increases and thus the oscillatory response improves. But looking at the poles of the system shown in fig. 10, when B_{dt} is increased by 25%, one pole moves towards right hand plane. This causes the instability of the system.

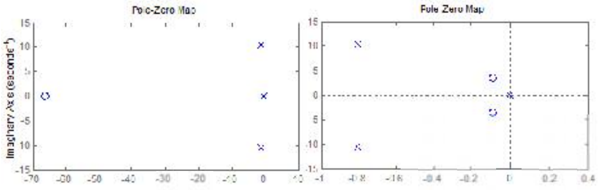

Fig.10 Pole –zero plot for the transfer functions of w_g/T_a and w_g/T_g with *50% increase in* B_{dt}

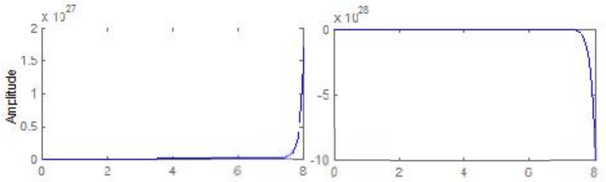

Fig.11 Step response for w_g/T_g *and* w_g/T_a *with 50% increase in* B_{dt}

Fig.11 shows the step responses of both the transfer function are unstable at B_{dt} more by 25%. Fig. 12 shows the transient response when fault occurred at 20s.

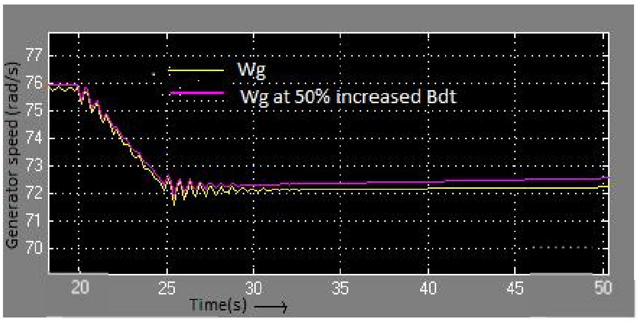

Fig.12 Generator speed when fault occur in drive train with actual B_{dt} and increased B_{dt} by 25%.

B. *Case 2: Change in shaft stiffness:*

The rotor inertia constant of the generator is much lower than that of the wind turbine. If the shaft stiffness of wind energy system is lesser, the drive train distortion may occur and results in the oscillation in the wind turbine torque during the faults.

During grid faults, the electrical torque is significantly reduced, and therefore the drive train system acts like a torsion spring that gets untwisted. Due to the torsion spring characteristic of the drive train, the transients are observed in generator speed. The large torque produced in the drive train may result in torsional oscillations among different sections of the wind turbine shaft system. In the severe cases, the drive shaft of a wind turbine generator may crack and even break. The fatigue accumulation when bearing a large torque repeatedly may reduce the life of the shaft.

Looking at the pole-zeroes shown in fig 13., there is no effect of change in stiffness constant on poles. But, zero in transfer function w_g/T_a is moving away from dominant poles as K_{dt} increases. So the oscillations are reduced.

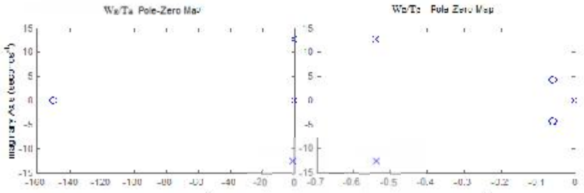

Fig.13Pole-zero plot for w_g/T_g *and* w_g/T_a *with 25% increase in* K_{dt}

C. *Case3 : Drive train gear ratio*

The gear ratio of drive train must be selected which produces the greatest amount of torque and does not exceed the maximum amount allowed by the generator.

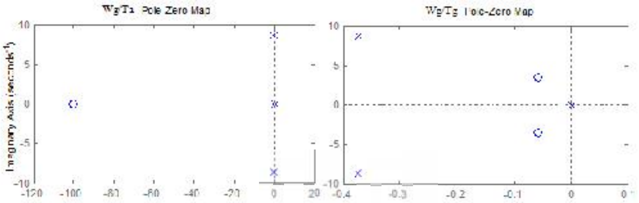

Fig. 14 Pole-zero plot for w_g/T_g *and* w_g/T_a *with* gear ratio increase by 20

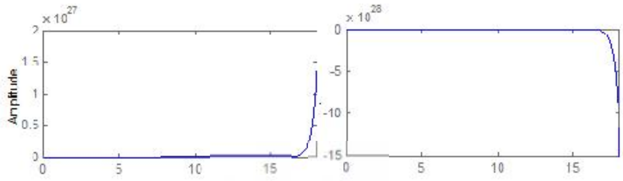

Fig15. Step response for w_g/T_g *and* w_g/T_a with gear ratio more by 20.

With increase in gear ratio the efficiency may be increased. But looking at the pole-zeroe, if we increase the gear ratio of one of the poles move towards right hand plane as shown in fig 14. And so the system move towards instability. With lesser gear ratio the system becomes more stable. So there are limitations to increase the gear ratio. Fig.15. shows that the increased gear ratio makes the step responses unstable. Fig. 16 shows that when gear ratio is high, the fault may lead the system to the instability.

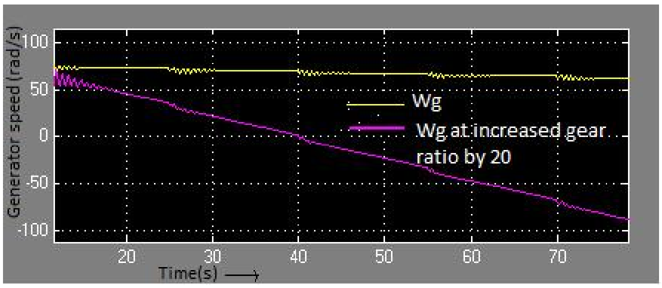

Fig.16 Generator speed when fault occur in drive train with actual gear ratio and increased gear ratio by 20.

D. *Case 4: Inertia constant*

Inertia constant of both turbine and generator has significant effect on transient stability. The inertia constant of the generator is much lower than that of the wind turbine. It leads to the drive train distortion result in the oscillation in the generator torque during the faults. The large total inertia constant make the system are more stable during power system disturbance or fault condition.

The poles-zero map shown in fig 17 shows that stability of the system can be increased with the increased generator inertia. If it is decreased one of the pole moves towards right hand plane and system becomes unstable. Fig.18 shows the step response of the w_g/T_a and w_g/T_g is unstable when inertia constant of generator decreased by 25%.

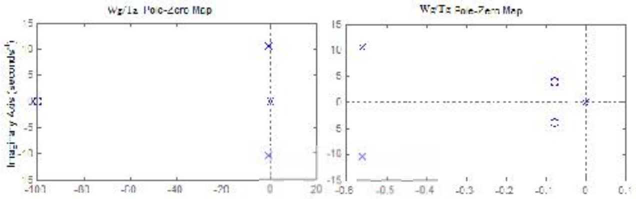

Fig. 17 Pole-zero plot for w_g/T_g and w_g/T_a with with rotor inertia less 25%.

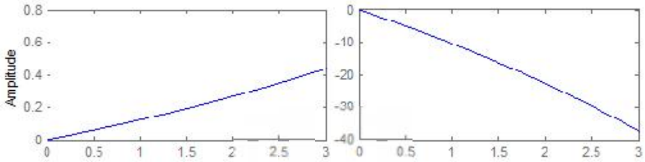

Fig.18 Step response *for w_g/T_g and w_g/T_a* with rotor inertia less 25%.

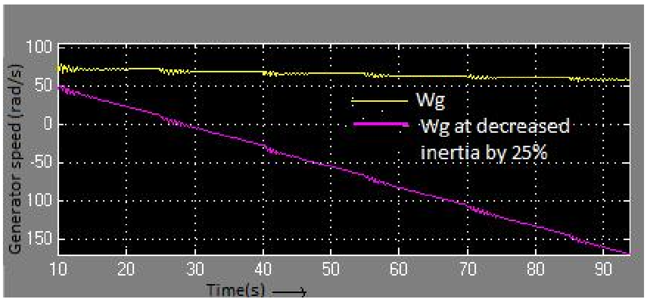

Fig.19 Generator speed when fault occur in drive train with actual and decreased generator inertia constant by 25%.

Fig.19 shows the instability of the system when fault occur at 25% decreased generator inertia constant. The faults cause transients also reflect in the power fluctuations. To minimize these oscillations and improve the performance, there are two ways. One way to mitigate these is to design perfect filtering while designing control system. Another way is to add a component, drive train stress damper to minimize these effects. It acts as a band pass filter.

V. CONCLUSION

In this work, by considering the frequent drive train faults the two mass model of wind energy system is modified. This model has been used for stability analysis by considering the sensitivity of the parameters generator inertia constant, spring constant, viscous friction coefficient and gear ratio. This study further is related with the physical design of the wind turbine. From the stability analysis it has been concluded that, the spring constant has less effect on stability. The increased viscous friction coefficient, gear ratio and decreased generator inertia can make the system unstable. Increased generator inertia constant improves the transient response. These observations are useful in to design fault tolerant control system for wind energy system.

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